Pedro A. Garrido
Superintendent of Schools
Phone (908) 245-1197
Fax (908) 245-1226

Roselle Park Public Schools
510 Chestnut Street
Roselle Park, New Jersey 07204

"A High Performing School District"

James Salvo
Director of Curriculum, Instruction, and Funded Programs
Phone (908) 245-6665 – ext. 1020
Fax (908) 245-6503

Home Instruction Packet for Pre-calculus CP

Name of Teacher and Class: K. Padilla / Pre-calculus CP

In this packet are materials and directions for Ms. Padilla's students.
This work will be collected and will be graded. The material will count towards 10% of the marking period grade. All assignments will be collected by submission via email following the schedule below. In the event that internet access is compromised, paper copies of the weekly assignments will be available on the school website at www.rphslibrary.org/home-instruction or can be physically picked up at the high school, all items will be collected when in-person classes resume.

I am available to support you during the hours 7:50am-2:50 pm to answer any of your questions. I will be responding to your emails within the hour.
You contact me at: Kpadilla@rpsd.org

Lesson: Title, Objective, What doing and how assessed.	Assignment Directions and how collected. Definitive due dates…
Week 1: Chapter 1 Review – Functions and their graphs	
Lesson 1 –1.1 & 1.3 – Rectangular coordinates & Linear equations in two variables. Pg.106: 1-6 & 29-42	Lesson 1 – Due 3/17 – Submit by emailing answers within a Microsoft word document or photograph of the completed assignment
Lesson 2 – 1.4 & 1.8 – Functions & Combinations of functions: Composite functions. Pg. 107: 43-54 & 81-86	Lesson 2 – Due 3/19 – Submit by emailing answers within a Microsoft word document or photograph of the completed assignment
Week 2: Chapter 2 Review – Polynomials and Rational Functions	
Lesson 1 – 2.1 & 2.2 – Quadratic functions and models & polynomial functions of higher degree. Pg. 190: 1-22	Lesson 1 – Due 3/24 – Submit by emailing answers within a Microsoft word document or photograph of the completed assignment
Lesson 2 – 2.3 & 2.4 – Polynomial and synthetic division & complex numbers. Pg. 190: 23-40	Lesson 2 – Due 3/26 – Submit by emailing answers within a Microsoft word document or photograph of the completed assignment
Week 3- Chapter 3 Review – Exponential and Logarithmic Functions	
Lesson 1 - 3.1 & 3.2 - Exponential functions and their graphs & Logarithmic functions and their graphs. Pg. 250: 1-58	Lesson 1 – Due 3/31 – Submit by emailing answers within a Microsoft word document or photograph of the completed assignment
Lesson 2 – 3.3 & 3.4 – Properties of logarithms & Exponential and logarithmic equations. Pg. 251: 59-104	Lesson 2 – Due 4/2– Submit by emailing answers within a Microsoft word document or photograph of the completed assignment

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to

1.1 Plotting Points in the Cartesian Plane In Exercises 1 and 2, plot the points.

1. $(5, 5), (-2, 0), (-3, 6), (-1, -7)$
2. $(0, 6), (8, 1), (5, -4), (-3, -3)$

Determining Quadrant(s) for a Point In Exercises 3 and 4, determine the quadrant(s) in which (x, y) could be located.

3. $x > 0$ and $y = -2$
4. $xy = 4$

5. **Plotting, Distance, and Midpoint** Plot the points $(-2, 6)$ and $(4, -3)$. Then find the distance between the points and the midpoint of the line segment joining the points.

6. **Sales** Barnes & Noble had annual sales of \$6.8 billion in 2013 and \$6.1 billion in 2015. Use the Midpoint Formula to estimate the sales in 2014. Assume that the annual sales follow a linear pattern. *(Source: Barnes & Noble, Inc.)*

1.2 Sketching the Graph of an Equation In Exercises 7–10, construct a table of values that consists of several points of the equation. Use the resulting solution points to sketch the graph of the equation.

7. $y = 3x - 5$
8. $y = -\frac{1}{2}x + 2$
9. $y = x^2 - 3x$
10. $y = 2x^2 - x - 9$

Finding x- and y-Intercepts In Exercises 11–14, find the x- and y-intercepts of the graph of the equation.

11. $y = 2x + 7$
12. $y = |x + 1| - 3$
13. $y = (x - 3)^2 - 4$
14. $y = x\sqrt{4 - x^2}$

Intercepts, Symmetry, and Graphing In Exercises 15–22, find any intercepts and test for symmetry. Then sketch the graph of the equation.

15. $y = -4x + 1$
16. $y = 5x - 6$
17. $y = 6 - x^2$
18. $y = x^2 - 12$
19. $y = x^3 + 5$
20. $y = -6 - x^3$
21. $y = \sqrt{x + 5}$
22. $y = |x| + 9$

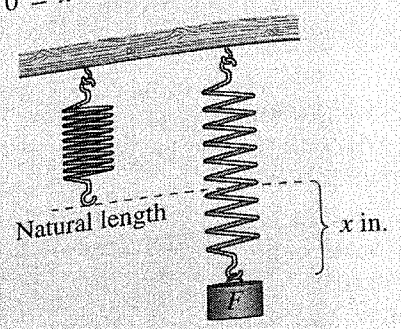
Sketching a Circle In Exercises 23–26, find the center and radius of the circle with the given equation. Then sketch the circle.

23. $x^2 + y^2 = 9$
24. $x^2 + y^2 = 4$
25. $(x + 2)^2 + y^2 = 16$
26. $x^2 + (y - 8)^2 = 81$

27. **Writing the Equation of a Circle** Write the standard form of the equation of the circle for which the endpoints of a diameter are $(0, 0)$ and $(4, -6)$.

28. **Physics** The force F (in pounds) required to stretch a spring x inches from its natural length (see figure) is

$$F = \frac{5}{4}x, \quad 0 \le x \le 20.$$



(a) Use the model to complete the table.

x	0	4	8	12	16	20
Force, F						

(b) Sketch a graph of the model.

(c) Use the graph to estimate the force necessary to stretch the spring 10 inches.

1.3 Graphing a Linear Equation In Exercises 29–32, find the slope and y-intercept (if possible) of the line. Sketch the line.

29. $y = -\frac{1}{2}x + 1$
30. $2x - 3y = 6$
31. $y = 1$
32. $x = -6$

Finding the Slope of a Line Through Two Points In Exercises 33 and 34, find the slope of the line passing through the pair of points.

33. $(5, -2), (-1, 4)$
34. $(-1, 6), (3, -2)$

Using the Point-Slope Form In Exercises 35 and 36, find the slope-intercept form of the equation of the line that has the given slope and passes through the given point. Sketch the line.

35. $m = \frac{1}{3}, \quad (6, -5)$
36. $m = -\frac{3}{4}, \quad (-4, -2)$

Finding an Equation of a Line In Exercises 37 and 38, find an equation of the line passing through the pair of points. Sketch the line.

37. $(-6, 4), (4, 9)$
38. $(-9, -3), (-3, -5)$

Finding Parallel and Perpendicular Lines In Exercises 39 and 40, find equations of the lines that pass through the given point and are (a) parallel to and (b) perpendicular to the given line.

39. $5x - 4y = 8, \quad (3, -2)$

40. $2x + 3y = 5, \quad (-8, 3)$

41. **Sales** A discount outlet offers a 20% discount on all items. Write a linear equation giving the sale price S for an item with a list price L.

42. **Hourly Wage** A manuscript translator charges a starting fee of \$50 plus \$2.50 per page translated. Write a linear equation for the amount A earned for translating p pages.

1.4 Testing for Functions Represented Algebraically In Exercises 43–46, determine whether the equation represents y as a function of x.

43. $16x - y^4 = 0$

44. $2x - y - 3 = 0$

45. $y = \sqrt{1 - x}$

46. $|y| = x + 2$

Evaluating a Function In Exercises 47 and 48, find each function value.

47. $f(x) = x^2 + 1$
    (a) $f(2)$
    (b) $f(-4)$
    (c) $f(t^2)$
    (d) $f(t + 1)$

48. $h(x) = |x - 2|$
    (a) $h(-4)$
    (b) $h(-2)$
    (c) $h(0)$
    (d) $h(-x + 2)$

Finding the Domain of a Function In Exercises 49 and 50, find the domain of the function.

49. $f(x) = \sqrt{25 - x^2}$

50. $h(x) = \dfrac{x}{x^2 - x - 6}$

Physics In Exercises 51 and 52, the velocity of a ball projected upward from ground level is given by $v(t) = -32t + 48$, where t is the time in seconds and v is the velocity in feet per second.

51. Find the velocity when $t = 1$.

52. Find the time when the ball reaches its maximum height. [*Hint:* Find the time when $v(t) = 0$.]

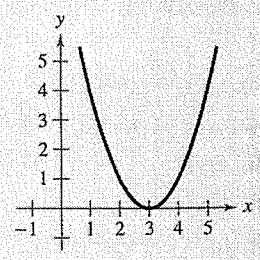
Evaluating a Difference Quotient In Exercises 53 and 54, find the difference quotient and simplify your answer.

53. $f(x) = 2x^2 + 3x - 1, \quad \dfrac{f(x + h) - f(x)}{h}, \quad h \neq 0$

54. $f(x) = x^3 - 5x^2 + x, \quad \dfrac{f(x + h) - f(x)}{h}, \quad h \neq 0$

1.5 Vertical Line Test for Functions In Exercises 55 and 56, use the Vertical Line Test to determine whether the graph represents y as a function of x. To print an enlarged copy of the graph, go to *MathGraphs.com*.

55.



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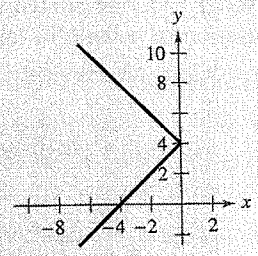


Finding the Zeros of a Function In Exercises 57 and 58, find the zeros of the function algebraically.

57. $f(x) = 3x^2 - 16x + 21$

58. $f(x) = 5x^2 + 4x - 1$

Describing Function Behavior In Exercises 59 and 60, use a graphing utility to graph the function and visually determine the open intervals on which the function is increasing, decreasing, or constant.

59. $f(x) = |x| + |x + 1|$

60. $f(x) = (x^2 - 4)^2$

Approximating Relative Minima or Maxima In Exercises 61 and 62, use a graphing utility to approximate (to two decimal places) any relative minima or maxima of the function.

61. $f(x) = -x^2 + 2x + 1$

62. $f(x) = x^3 - 4x^2 - 1$

Average Rate of Change of a Function In Exercises 63 and 64, find the average rate of change of the function from x_1 to x_2.

63. $f(x) = -x^2 + 8x - 4, \quad x_1 = 0, x_2 = 4$

64. $f(x) = x^3 + 2x + 1, \quad x_1 = 1, x_2 = 3$

Even, Odd, or Neither? In Exercises 65 and 66, determine whether the function is even, odd, or neither. Then describe the symmetry.

65. $f(x) = x^4 - 20x^2$

66. $f(x) = 2x\sqrt{x^2 + 3}$

1.6 Writing a Linear Function In Exercises 67 and 68, (a) write the linear function f that has the given function values and (b) sketch the graph of the function.

67. $f(2) = -6, \quad f(-1) = 3$

68. $f(0) = -5, \quad f(4) = -8$

Graphing a Function In Exercises 69 and 70, sketch the graph of the function.

69. $g(x) = [\![x]\!] - 2$

70. $f(x) = \begin{cases} 5x - 3, & x \geq -1 \\ -4x + 5, & x < -1 \end{cases}$

1.7 Describing Transformations In Exercises 71–80, h is related to one of the parent functions described in this chapter. (a) Identify the parent function f. (b) Describe the sequence of transformations from f to h. (c) Sketch the graph of h. (d) Use function notation to write h in terms of f.

71. $h(x) = x^2 - 9$
72. $h(x) = (x - 2)^3 + 2$
73. $h(x) = -\sqrt{x} + 4$
74. $h(x) = |x + 3| - 5$
75. $h(x) = -(x + 2)^2 + 3$
76. $h(x) = \frac{1}{2}(x - 1)^2 - 2$
77. $h(x) = -[\![x]\!] + 6$
78. $h(x) = -\sqrt{x + 1} + 9$
79. $h(x) = 5[\![x - 9]\!]$
80. $h(x) = -\frac{1}{3}x^3$

1.8 Finding Arithmetic Combinations of Functions In Exercises 81 and 82, find (a) $(f + g)(x)$, (b) $(f - g)(x)$, (c) $(fg)(x)$, and (d) $(f/g)(x)$. What is the domain of f/g?

81. $f(x) = x^2 + 3, \quad g(x) = 2x - 1$
82. $f(x) = x^2 - 4, \quad g(x) = \sqrt{3 - x}$

Finding Domains of Functions and Composite Functions In Exercises 83 and 84, find (a) $f \circ g$ and (b) $g \circ f$. Find the domain of each function and of each composite function.

83. $f(x) = \frac{1}{3}x - 3, \quad g(x) = 3x + 1$
84. $f(x) = x^3 - 4, \quad g(x) = \sqrt[3]{x + 7}$

Retail In Exercises 85 and 86, the price of a washing machine is x dollars. The function

$$f(x) = x - 100$$

gives the price of the washing machine after a \$100 rebate. The function

$$g(x) = 0.95x$$

gives the price of the washing machine after a 5% discount.

85. Find and interpret $(f \circ g)(x)$.
86. Find and interpret $(g \circ f)(x)$.

1.9 Finding an Inverse Function Informally In Exercises 87 and 88, find the inverse function of f informally. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

87. $f(x) = 3x + 8$
88. $f(x) = \dfrac{x - 4}{5}$

Applying the Horizontal Line Test In Exercises 89 and 90, use a graphing utility to graph the function, and use the Horizontal Line Test to determine whether the function has an inverse function.

89. $f(x) = (x - 1)^2$
90. $h(t) = \dfrac{2}{t - 3}$

Finding and Analyzing Inverse Functions In Exercises 91 and 92, (a) find the inverse function of f, (b) graph both f and f^{-1} on the same set of coordinate axes, (c) describe the relationship between the graphs of f and f^{-1}, and (d) state the domains and ranges of f and f^{-1}.

91. $f(x) = \frac{1}{2}x - 3$
92. $f(x) = \sqrt{x + 1}$

Restricting the Domain In Exercises 93 and 94, restrict the domain of the function f to an interval on which the function is increasing, and find f^{-1} on that interval.

93. $f(x) = 2(x - 4)^2$
94. $f(x) = |x - 2|$

1.10

95. **Agriculture** The ordered pairs below give the amount B (in millions of pounds) of beef produced on private farms each year from 2007 through 2014. *(Spreadsheet at LarsonPrecalculus.com)* (*Source: United States Department of Agriculture*)

DATA

(2007, 102.7)	(2010, 84.2)	(2013, 70.4)
(2008, 95.9)	(2011, 75.0)	(2014, 67.9)
(2009, 90.2)	(2012, 76.3)	

(a) Use a graphing utility to create a scatter plot of the data. Let t represent the year, with $t = 7$ corresponding to 2007.

(b) Use the *regression* feature of the graphing utility to find the equation of the least squares regression line that fits the data. Then graph the model and the scatter plot you found in part (a) in the same viewing window. How closely does the model represent the data?

96. **Travel Time** The travel time between two cities is inversely proportional to the average speed. A train travels between the cities in 3 hours at an average speed of 65 miles per hour. How long does it take to travel between the cities at an average speed of 80 miles per hour?

97. **Cost** The cost of constructing a wooden box with a square base varies jointly as the height of the box and the square of the width of the box. Constructing a box of height 16 inches and of width 6 inches costs \$28.80. How much does it cost to construct a box of height 14 inches and of width 8 inches?

Exploration

True or False? In Exercises 98 and 99, determine whether the statement is true or false. Justify your answer.

98. Relative to the graph of $f(x) = \sqrt{x}$, the graph of the function $h(x) = -\sqrt{x + 9} - 13$ is shifted 9 units to the left and 13 units down, then reflected in the x-axis.

99. If f and g are two inverse functions, then the domain of g is equal to the range of f.

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

2.1 Sketching Graphs of Quadratic Functions In Exercises 1 and 2, sketch the graph of each quadratic function and compare it with the graph of $y = x^2$.

1. (a) $g(x) = -2x^2$ (b) $h(x) = x^2 + 2$
2. (a) $h(x) = (x - 3)^2$ (b) $k(x) = \frac{1}{2}x - 1$

Using Standard Form to Graph a Parabola In Exercises 3–8, write the quadratic function in standard form and sketch its graph. Identify the vertex, axis of symmetry, and x-intercept(s).

3. $g(x) = x^2 - 2x$
4. $f(x) = x^2 + 8x + 10$
5. $h(x) = 3 + 4x - x^2$
6. $f(t) = -2t^2 + 4t + 1$
7. $h(x) = 4x^2 + 4x + 13$
8. $f(x) = \frac{1}{3}(x^2 + 5x - 4)$

9. **Geometry** The perimeter of a rectangle is 1000 meters.
   (a) Write the width y as a function of the length x. Use the result to write the area A as a function of x.
   (b) Of all possible rectangles with perimeters of 1000 meters, find the dimensions of the one with the maximum area.

10. **Minimum Cost** A soft-drink manufacturer has a daily production cost of $C = 70{,}000 - 120x + 0.055x^2$, where C is the total cost (in dollars) and x is the number of units produced. How many units should they produce each day to yield a minimum cost?

2.2 Sketching a Transformation of a Monomial Function In Exercises 11 and 12, sketch the graphs of $y = x^n$ and the transformation.

11. $y = x^4, \quad f(x) = 6 - x^4$
12. $y = x^5, \quad f(x) = \frac{1}{2}x^5 + 3$

Applying the Leading Coefficient Test In Exercises 13–16, describe the left-hand and right-hand behavior of the graph of the polynomial function.

13. $f(x) = -2x^2 - 5x + 12$
14. $f(x) = 4x - \frac{1}{2}x^3$
15. $g(x) = -3x^3 - 8x^4 + x^5$
16. $h(x) = 5 + 9x^6 - 6x^5$

Sketching the Graph of a Polynomial Function In Exercises 17–20, sketch the graph of the function by (a) applying the Leading Coefficient Test, (b) finding the real zeros of the polynomial, (c) plotting sufficient solution points, and (d) drawing a continuous curve through the points.

17. $g(x) = 2x^3 + 4x^2$
18. $h(x) = 3x^2 - x^4$
19. $f(x) = -x^3 + x^2 - 2$
20. $f(x) = x(x^3 + x^2 - 5x + 3)$

Using the Intermediate Value Theorem In Exercises 21 and 22, (a) use the Intermediate Value Theorem and the *table* feature of a graphing utility to find intervals one unit in length in which the polynomial function is guaranteed to have a zero. (b) Adjust the table to approximate the zeros of the function to the nearest thousandth. Use the *zero* or *root* feature of the graphing utility to verify your results.

21. $f(x) = 3x^3 - x^2 + 3$
22. $f(x) = x^4 - 5x - 1$

2.3 Long Division of Polynomials In Exercises 23 and 24, use long division to divide.

23. $\dfrac{30x^2 - 3x + 8}{5x - 3}$
24. $\dfrac{5x^3 - 21x^2 - 25x - 4}{x^2 - 5x - 1}$

Using Synthetic Division In Exercises 25 and 26, use synthetic division to divide.

25. $\dfrac{2x^3 - 25x^2 + 66x + 48}{x - 8}$
26. $\dfrac{x^4 - 2x^2 + 9x}{x + 3}$

Factoring a Polynomial In Exercises 27 and 28, (a) verify the given factor(s) of $f(x)$, (b) find the remaining factors of $f(x)$, (c) use your results to write the complete factorization of $f(x)$, (d) list all real zeros of f, and (e) confirm your results by using a graphing utility to graph the function.

	Function	Factor(s)
27.	$f(x) = 2x^3 + 11x^2 - 21x - 90$	$(x + 6)$
28.	$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$	$(x + 2), (x - 3)$

2.4 Writing a Complex Number in Standard Form In Exercises 29 and 30, write the complex number in standard form.

29. $4 + \sqrt{-9}$
30. $-5i + i^2$

Performing Operations with Complex Numbers In Exercises 31–34, perform the operation and write the result in standard form.

31. $(6 - 4i) + (-9 + i)$
32. $(7 - 2i) - (3 - 8i)$
33. $-3i(-2 + 5i)$
34. $(4 + i)(3 - 10i)$

Quotient of Complex Numbers in Standard Form In Exercises 35 and 36, write the quotient in standard form.

35. $\dfrac{4}{1 - 2i}$
36. $\dfrac{3 + 2i}{5 + i}$

Performing Operations with Complex Numbers **In Exercises 37 and 38, perform the operation and write the result in standard form.**

37. $\dfrac{4}{2-3i} + \dfrac{2}{1+i}$

38. $\dfrac{1}{2+i} - \dfrac{5}{1+4i}$

Complex Solutions of a Quadratic Equation **In Exercises 39 and 40, use the Quadratic Formula to solve the quadratic equation.**

39. $x^2 - 2x + 10 = 0$

40. $6x^2 + 3x + 27 = 0$

2.5 Zeros of Polynomial Functions **In Exercises 41 and 42, determine the number of zeros of the polynomial function.**

41. $g(x) = x^2 - 2x - 8$

42. $h(t) = t^2 - t^5$

Using the Rational Zero Test **In Exercises 43 and 44, find the rational zeros of the function.**

43. $f(x) = 4x^3 - 27x^2 + 11x + 42$

44. $f(x) = x^4 + x^3 - 11x^2 + x - 12$

Finding the Zeros of a Polynomial Function **In Exercises 45 and 46, write the polynomial as the product of linear factors and list all the zeros of the function.**

45. $g(x) = x^3 - 7x^2 + 36$

46. $f(x) = x^4 + 8x^3 + 8x^2 - 72x - 153$

47. **Using Descartes's Rule of Signs** Use Descartes's Rule of Signs to determine the possible numbers of positive and negative real zeros of $h(x) = -2x^5 + 4x^3 - 2x^2 + 5$.

48. **Verifying Upper and Lower Bounds** Use synthetic division to verify the upper and lower bounds of the real zeros of $f(x) = 4x^3 - 3x^2 + 4x - 3$.

(a) Upper: $x = 1$ (b) Lower: $x = -\frac{1}{4}$

2.6 Finding Domain and Asymptotes **In Exercises 49 and 50, find the domain and the vertical and horizontal asymptotes of the graph of the rational function.**

49. $f(x) = \dfrac{3x}{x+10}$

50. $f(x) = \dfrac{8}{x^2 - 10x + 24}$

Sketching the Graph of a Rational Function **In Exercises 51–58, (a) state the domain of the function, (b) identify all intercepts, (c) find any asymptotes, and (d) plot additional solution points as needed to sketch the graph of the rational function.**

51. $f(x) = \dfrac{4}{x}$

52. $h(x) = \dfrac{x-4}{x-7}$

53. $f(x) = \dfrac{x}{x^2 - 16}$

54. $f(x) = \dfrac{-8x}{x^2 + 4}$

55. $f(x) = \dfrac{6x^2 - 11x + 3}{3x^2 - x}$

56. $f(x) = \dfrac{6x^2 - 7x + 2}{4x^2 - 1}$

57. $f(x) = \dfrac{2x^3}{x^2 + 1}$

58. $f(x) = \dfrac{2x^2 + 2}{x + 1}$

59. **Seizure of Illegal Drugs** The cost C (in millions of dollars) for the federal government to seize $p\%$ of an illegal drug as it enters the country is given by

$$C = \frac{528p}{100 - p}, \quad 0 \le p \le 100.$$

(a) Use a graphing utility to graph the cost function.

(b) Find the costs of seizing 25%, 50%, and 75% of the drug.

(c) According to the model, it is possible to seize 100% of the drug? Explain.

60. **Page Design** A page that is x inches wide and y inches high contains 30 square inches of print. The top and bottom margins are each 2 inches deep, and the margins on each side are 2 inches wide.

(a) Write a function for the total area A of the page in terms of x.

(b) Determine the domain of the function based of the physical constraints of the problem.

(c) Use a graphing utility to graph the area function and approximate the dimensions of the page that use the least amount of paper.

2.7 Solving an Inequality **In Exercises 61–64, solve the inequality. Then graph the solution set.**

61. $12x^2 + 5x < 2$

62. $x^3 - 16x \ge 0$

63. $\dfrac{2}{x+1} \ge \dfrac{3}{x-1}$

64. $\dfrac{x^2 - 9x + 20}{x} < 0$

65. **Biology** A biologist introduces 200 ladybugs into a crop field. The population P of the ladybugs can be approximated by the model

$$P = \frac{1000(1 + 3t)}{5 + t}$$

where t is the time in days. Find the time required for the population to increase to at least 2000 ladybugs.

Exploration

True or False? **In Exercises 66 and 67, determine whether the statement is true or false. Justify your answer.**

66. A fourth-degree polynomial with real coefficients can have -5, $-8i$, $4i$, and 5 as its zeros.

67. The domain of a rational function can never be the set of all real numbers.

68. **Writing** Describe what is meant by an asymptote of a graph.

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

3.1 Evaluating an Exponential Function In Exercises 1–6, evaluate the function at the given value of x. Round your result to three decimal places.

1. $f(x) = 0.3^x, \quad x = 1.5$
2. $f(x) = 30^x, \quad x = \sqrt{3}$
3. $f(x) = 2^x, \quad x = \frac{2}{3}$
4. $f(x) = \left(\frac{1}{2}\right)^{2x}, \quad x = \pi$
5. $f(x) = 7(0.2^x), \quad x = -\sqrt{11}$
6. $f(x) = -14(5^x), \quad x = -0.8$

Graphing an Exponential Function In Exercises 7–12, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.

7. $f(x) = 4^{-x} + 4$
8. $f(x) = 2.65^{x-1}$
9. $f(x) = 5^{x-2} + 4$
10. $f(x) = 2^{x-6} - 5$
11. $f(x) = \left(\frac{1}{2}\right)^{-x} + 3$
12. $f(x) = \left(\frac{1}{8}\right)^{x+2} - 5$

Using a One-to-One Property In Exercises 13–16, use a One-to-One Property to solve the equation for x.

13. $\left(\frac{1}{3}\right)^{x-3} = 9$
14. $3^{x+3} = \frac{1}{81}$
15. $e^{3x-5} = e^7$
16. $e^{8-2x} = e^{-3}$

Transforming the Graph of an Exponential Function In Exercises 17–20, describe the transformation of the graph of f that yields the graph of g.

17. $f(x) = 5^x, \quad g(x) = 5^x + 1$
18. $f(x) = 6^x, \quad g(x) = 6^{x+1}$
19. $f(x) = 3^x, \quad g(x) = 1 - 3^x$
20. $f(x) = \left(\frac{1}{2}\right)^x, \quad g(x) = -\left(\frac{1}{2}\right)^{x+2}$

Evaluating the Natural Exponential Function In Exercises 21–24, evaluate $f(x) = e^x$ at the given value of x. Round your result to three decimal places.

21. $x = 3.4$
22. $x = -2.5$
23. $x = \frac{3}{5}$
24. $x = \frac{2}{7}$

Graphing a Natural Exponential Function In Exercises 25–28, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.

25. $h(x) = e^{-x/2}$
26. $h(x) = 2 - e^{-x/2}$
27. $f(x) = e^{x+2}$
28. $s(t) = 4e^{t-1}$

29. **Waiting Times** The average time between new posts on a message board is 3 minutes. The probability F of waiting less than t minutes until the next post is approximated by the model $F(t) = 1 - e^{-t/3}$. A message has just been posted. Find the probability that the next post will be within (a) 1 minute, (b) 2 minutes, and (c) 5 minutes.

30. **Depreciation** After t years, the value V of a car that originally cost \$23,970 is given by $V(t) = 23{,}970\left(\frac{3}{4}\right)^t$.
    (a) Use a graphing utility to graph the function.
    (b) Find the value of the car 2 years after it was purchased.
    (c) According to the model, when does the car depreciate most rapidly? Is this realistic? Explain.
    (d) According to the model, when will the car have no value?

Compound Interest In Exercises 31 and 32, complete the table by finding the balance A when P dollars is invested at rate r for t years and compounded n times per year.

n	1	2	4	12	365	Continuous
A						

31. $P = \$5000, \quad r = 3\%, \quad t = 10$ years
32. $P = \$4500, \quad r = 2.5\%, \quad t = 30$ years

3.2 Writing a Logarithmic Equation In Exercises 33–36, write the exponential equation in logarithmic form. For example, the logarithmic form of $2^3 = 8$ is $\log_2 8 = 3$.

33. $3^3 = 27$
34. $25^{3/2} = 125$
35. $e^{0.8} = 2.2255\ldots$
36. $e^0 = 1$

Evaluating a Logarithm In Exercises 37–40, evaluate the logarithm at the given value of x without using a calculator.

37. $f(x) = \log x, \quad x = 1000$
38. $g(x) = \log_9 x, \quad x = 3$
39. $g(x) = \log_2 x, \quad x = \frac{1}{4}$
40. $f(x) = \log_3 x, \quad x = \frac{1}{81}$

Using a One-to-One Property In Exercises 41–44, use a One-to-One Property to solve the equation for x.

41. $\log_4(x + 7) = \log_4 14$
42. $\log_8(3x - 10) = \log_8 5$
43. $\ln(x + 9) = \ln 4$
44. $\log(3x - 2) = \log 7$

Sketching the Graph of a Logarithmic Function In Exercises 45–48, find the domain, x-intercept, and vertical asymptote of the logarithmic function and sketch its graph.

45. $g(x) = \log_7 x$
46. $f(x) = \log \frac{x}{3}$
47. $f(x) = 4 - \log(x + 5)$
48. $f(x) = \log(x - 3) + 1$

Evaluating a Logarithmic Function **In Exercises 49–52, use a calculator to evaluate the function at the given value of x. Round your result to three decimal places, if necessary.**

49. $f(x) = \ln x,\ x = 22.6$ **50.** $f(x) = \ln x,\ x = e^{-12}$

51. $f(x) = \frac{1}{2}\ln x,\ x = \sqrt{e}$

52. $f(x) = 5\ln x,\ x = 0.98$

Graphing a Natural Logarithmic Function **In Exercises 53–56, find the domain, x-intercept, and vertical asymptote of the logarithmic function and sketch its graph.**

53. $f(x) = \ln x + 6$ **54.** $f(x) = \ln x - 5$

55. $h(x) = \ln(x - 6)$ **56.** $f(x) = \ln(x + 4)$

57. Astronomy The formula $M = m - 5\log(d/10)$ gives the distance d (in parsecs) from Earth to a star with apparent magnitude m and absolute magnitude M. The star Rasalhague has an apparent magnitude of 2.08 and an absolute magnitude of 1.3. Find the distance from Earth to Rasalhague.

58. Snow Removal The number of miles s of roads cleared of snow is approximated by the model

$$s = 25 - \frac{13\ln(h/12)}{\ln 3}, \quad 2 \le h \le 15$$

where h is the depth (in inches) of the snow. Use this model to find s when $h = 10$ inches.

3.3 Using the Change-of-Base Formula **In Exercises 59–62, evaluate the logarithm using the change-of-base formula (a) with common logarithms and (b) with natural logarithms. Round your results to three decimal places.**

59. $\log_2 6$ **60.** $\log_{12} 200$

61. $\log_{1/2} 5$ **62.** $\log_4 0.75$

Using Properties of Logarithms **In Exercises 63–66, use the properties of logarithms to write the logarithm in terms of $\log_2 3$ and $\log_2 5$.**

63. $\log_2 \frac{5}{3}$ **64.** $\log_2 45$

65. $\log_2 \frac{9}{5}$ **66.** $\log_2 \frac{20}{9}$

Expanding a Logarithmic Expression **In Exercises 67–72, use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive.)**

67. $\log 7x^2$ **68.** $\log 11x^3$

69. $\log_3 \dfrac{9}{\sqrt{x}}$ **70.** $\log_7 \dfrac{\sqrt[3]{x}}{19}$

71. $\ln x^2y^2z$ **72.** $\ln\left(\dfrac{y-1}{3}\right)^2,\quad y > 1$

Condensing a Logarithmic Expression **In Exercises 73–78, condense the expression to the logarithm of a single quantity.**

73. $\ln 7 + \ln x$

74. $\log_2 y - \log_2 3$

75. $\log x - \frac{1}{2}\log y$

76. $3\ln x + 2\ln(x + 1)$

77. $\frac{1}{2}\log_3 x - 2\log_3(y + 8)$

78. $5\ln(x - 2) - \ln(x + 2) - 3\ln x$

79. Climb Rate The time t (in minutes) for a small plane to climb to an altitude of h feet is modeled by

$$t = 50\log[18{,}000/(18{,}000 - h)]$$

where 18,000 feet is the plane's absolute ceiling.

(a) Determine the domain of the function in the context of the problem.

(b) Use a graphing utility to graph the function and identify any asymptotes.

(c) As the plane approaches its absolute ceiling, what can be said about the time required to increase its altitude?

(d) Find the time it takes for the plane to climb to an altitude of 4000 feet.

80. Human Memory Model Students in a learning theory study took an exam and then retested monthly for 6 months with an equivalent exam. The data obtained in the study are given by the ordered pairs (t, s), where t is the time (in months) after the initial exam and s is the average score for the class. Use the data to find a logarithmic equation that relates t and s.

(1, 84.2), (2, 78.4), (3, 72.1),
(4, 68.5), (5, 67.1), (6, 65.3)

3.4 Solving a Simple Equation **In Exercises 81–86, solve for x.**

81. $5^x = 125$

82. $6^x = \frac{1}{216}$

83. $e^x = 3$

84. $\log x - \log 5 = 0$

85. $\ln x = 4$

86. $\ln x = -1.6$

Solving an Exponential Equation **In Exercises 87–90, solve the exponential equation algebraically. Approximate the result to three decimal places.**

87. $e^{4x} = e^{x^2+3}$

88. $e^{3x} = 25$

89. $2^x - 3 = 29$

90. $e^{2x} - 6e^x + 8 = 0$

Solving a Logarithmic Equation **In Exercises 91–98, solve the logarithmic equation algebraically. Approximate the result to three decimal places.**

91. $\ln 3x = 8.2$

92. $4 \ln 3x = 15$

93. $\ln x + \ln(x - 3) = 1$

94. $\ln(x + 2) - \ln x = 2$

95. $\log_8(x - 1) = \log_8(x - 2) - \log_8(x + 2)$

96. $\log_6(x + 2) - \log_6 x = \log_6(x + 5)$

97. $\log(1 - x) = -1$

98. $\log(-x - 4) = 2$

Using Technology **In Exercises 99–102, use a graphing utility to graphically solve the equation. Approximate the result to three decimal places. Verify your result algebraically.**

99. $25e^{-0.3x} = 12$

100. $2 = 5 - e^{x+7}$

101. $2 \ln(x + 3) - 3 = 0$

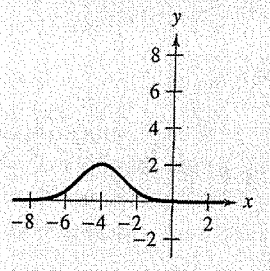
102. $2 \ln x - \ln(3x - 1) = 0$

103. Compound Interest You deposit \$8500 in an account that pays 1.5% interest, compounded continuously. How long will it take for the money to triple?

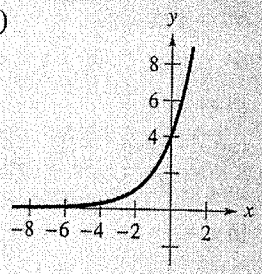
104. Meteorology The speed of the wind S (in miles per hour) near the center of a tornado and the distance d (in miles) the tornado travels are related by the model $S = 93 \log d + 65$. On March 18, 1925, a large tornado struck portions of Missouri, Illinois, and Indiana with a wind speed at the center of about 283 miles per hour. Approximate the distance traveled by this tornado.

3.5 Matching a Function with Its Graph **In Exercises 105–110, match the function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]**

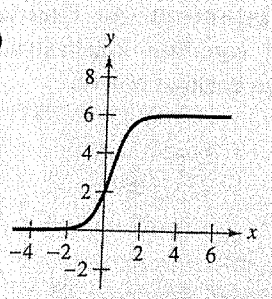
(a)



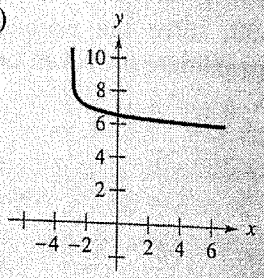
(b)



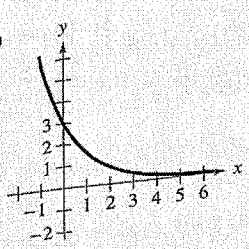
(c)



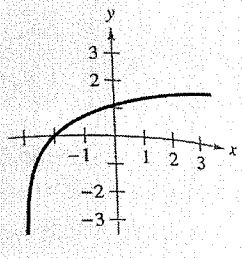
(d)



(e)



(f)



105. $y = 3e^{-2x/3}$

106. $y = 4e^{2x/3}$

107. $y = \ln(x + 3)$

108. $y = 7 - \log(x + 3)$

109. $y = 2e^{-(x+4)^2/3}$

110. $y = \dfrac{6}{1 + 2e^{-2x}}$

111. Finding an Exponential Model Find the exponential model $y = ae^{bx}$ that fits the points $(0, 2)$ and $(4, 3)$.

112. Wildlife Population A species of bat is in danger of becoming extinct. Five years ago, the total population of the species was 2000. Two years ago, the total population of the species was 1400. What was the total population of the species one year ago?

113. Test Scores The test scores for a biology test follow the normal distribution

$$y = 0.0499e^{-(x-71)^2/128}, \quad 40 \le x \le 100$$

where x is the test score. Use a graphing utility to graph the equation and estimate the average test score.

114. Typing Speed In a typing class, the average number N of words per minute typed after t weeks of lessons is

$$N = 157/(1 + 5.4e^{-0.12t}).$$

Find the time necessary to type (a) 50 words per minute and (b) 75 words per minute.

115. Sound Intensity The relationship between the number of decibels β and the intensity of a sound I (in watts per square meter) is

$$\beta = 10 \log(I/10^{-12}).$$

Find the intensity I for each decibel level β.

(a) $\beta = 60$ (b) $\beta = 135$ (c) $\beta = 1$

Exploration

116. Graph of an Exponential Function Consider the graph of $y = e^{kt}$. Describe the characteristics of the graph when k is positive and when k is negative.

True or False? **In Exercises 117 and 118, determine whether the equation is true or false. Justify your answer.**

117. $\log_b b^{2x} = 2x$

118. $\ln(x + y) = \ln x + \ln y$