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Home Instruction Packet for Calculus Honors

Mrs. Clausi

<table>
<tr><td colspan="2">In this packet are materials and directions…..

Videos to follow are located on www.calculus.flippedmath.com
Follow through the notes and examples on the videos, then complete the practice assignments. All of the work must be shown in the packet.

This work will be collected by the teacher when we return to school. I will also check work during the home instruction period through email or remind. This work will be graded and counted towards their marking period grade.</td></tr>
<tr><td colspan="2">I am available to support you during the hours 7:50am-2:50 pm to answer any of your questions. I will be responding to your emails within the hour.

You contact me at: jclausi@rpsd.org and using our class remind app</td></tr>
<tr><td>Lesson: Calculus Review

Students will complete a review packet of Calculus concepts, through watching videos, taking notes, and completing practice problems.</td><td>Directions for packet:
Answer all questions and show all work in your packet.

Directions for emailing answers:
1. Email/Remind me your answers for the week according to the schedule below. Take note of due dates and due times.
2. You may type out your final answers in the email OR email me a picture of the page(s) with your work and answers.</td></tr>
<tr><td>Week 1-

Lesson 1: Limits Graphically

Lesson 2: 1.2 Limits Analytically

Lesson 3: 1.2 Limits Analytically continued

Week 2-

Lesson 1: 3.1 Power Rule

Lesson 2: 3.1 Power Rule continued

Lesson 3: 3.2 Product & Quotient Rule</td><td>Watch the lesson video and take notes. Complete the practice problems for each section.</td></tr>
</table>

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Week 3 – **Lesson 1: 3.2 Product & Quotient Rule continued** **Lesson 2: 6.1 Implicit Differentiation** **Lesson 2: 6.1 Implicit Differentiation Continued**	Watch the lesson video and take notes. Complete the practice problems for each section.

www. Calculus. flippedmath. com

Calculus

Write your questions and thoughts here!

1.1 Limits Graphically

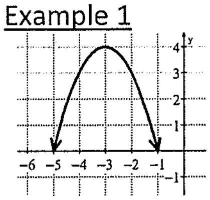
Name:________________________

Notes

What is a limit?

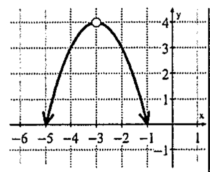
A ***limit*** is the ___________ a function _______________ from *both* the left and the right side of a given ____________.

Example 1



$f(-3) =$

$\lim_{x \to -3} f(x) =$



$f(-3) =$

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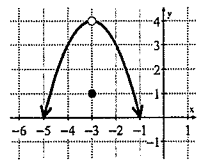

$f(-3) =$

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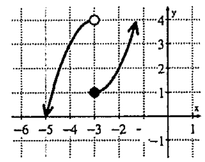
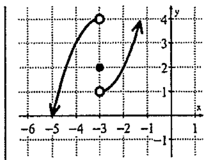


$f(-3) =$

$\lim_{x \to -3} f(x) =$



$f(-3) =$

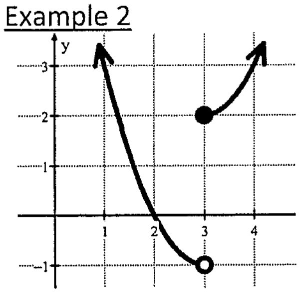
$\lim_{x \to -3} f(x) =$

Limit: (*geeky math definition for Mr. Kelly*)
Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = R$.

What is a *one-sided* limit?

A ***one-sided limit*** is the ____________ a function approaches as you approach a given ____________ from either the _______ or _______ side.

Example 2



"The limit of f as x approaches 3 from the left side is -1." $\lim_{x \to 3^-} f(x) = -1$

"The limit of f as x approaches 3 from the right side is 2." $\lim_{x \to 3^+} f(x) = 2$

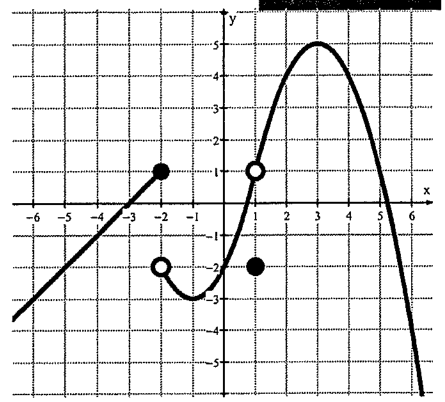
1.1 Limits Graphically

Write your questions and thoughts here!

Notes

Example 3

a. $\lim\limits_{x\to -2^-} f(x)=$	b. $\lim\limits_{x\to -2^+} f(x)=$	c. $\lim\limits_{x\to -2} f(x)=$
d. $\lim\limits_{x\to 1} f(x)=$	e. $\lim\limits_{x\to 0} f(x)=$	f. $\lim\limits_{x\to 3^-} f(x)=$
g. $\lim\limits_{x\to -1} f(x)=$	h. $\lim\limits_{x\to -3} f(x)=$	i. $f(-2)=$
j. $f(1)=$		



When does a limit not exist?

1.
2.
3.

Example 4

Sketch a graph of a function g that satisfies all of the following conditions.

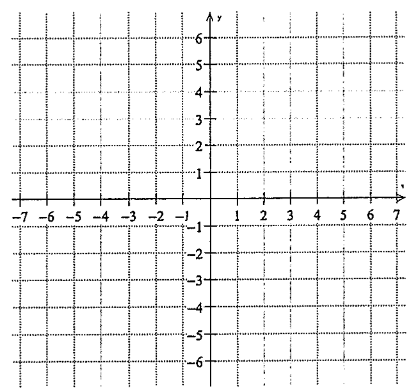
a. $g(3)=-1$

b. $\lim\limits_{x\to 3} g(x)=4$

c. $\lim\limits_{x\to -2^+} g(x)=1$

d. g is increasing on $-2<x<3$

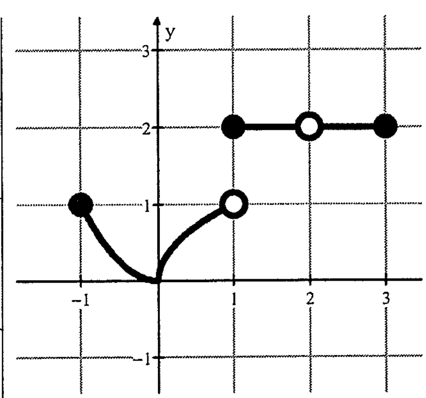
e. $\lim\limits_{x\to -2^-} g(x)>\lim\limits_{x\to -2^+} g(x)$



Example 5

Write **T** (true) or **F** (false) under each statement. Use the graph on the right.

a. $\lim\limits_{x\to -1^+} f(x)=1$	b. $\lim\limits_{x\to 2} f(x)=2$	c. $\lim\limits_{x\to 1^-} f(x)=1$
d. $\lim\limits_{x\to 1^+} f(x)=2$	e. $\lim\limits_{x\to 1} f(x)=$ does not exist	
f. $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x)$	g. $\lim\limits_{x\to 2} f(x)=$ does not exist	



Now summarize what you learned!

1.1 Limits Graphically

Calculus Name: ____________________________ **Practice**

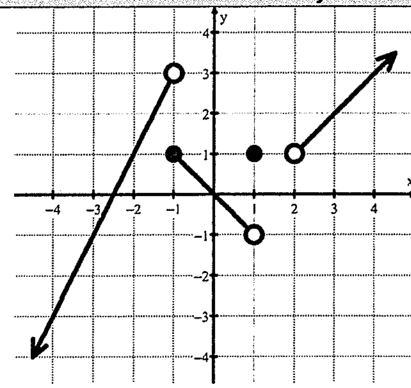
For 1-5, give the value of each statement. If the value does not exist, write *"does not exist"* or *"undefined."*

1.

a. $\lim_{x\to -1^-} f(x) =$ b. $f(1) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 2^+} f(x) =$ e. $f(-1) =$ f. $f(2) =$

g. $\lim_{x\to -1^+} f(x) =$ h. $\lim_{x\to 1^-} f(x) =$ i. $\lim_{x\to 2} f(x) =$

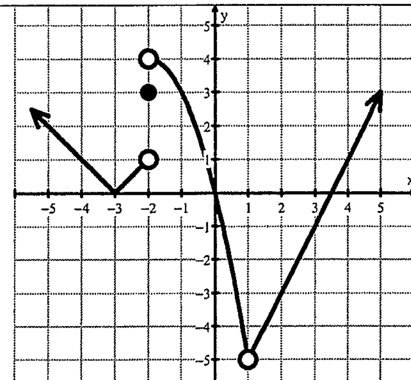


2.

a. $\lim_{x\to -3} f(x) =$ b. $f(1) =$ c. $\lim_{x\to 1} f(x) =$

d. $\lim_{x\to -2^+} f(x) =$ e. $f(3) =$ f. $\lim_{x\to -2^-} f(x) =$

g. $\lim_{x\to -2} f(x) =$ h. $f(-2) =$ i. $f(4) =$

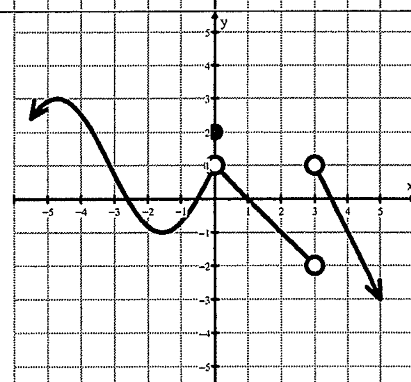


3.

a. $\lim_{x\to 3^+} f(x) =$ b. $f(3) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 3} f(x) =$ e. $f(0) =$ f. $\lim_{x\to 3^-} f(x) =$

g. $\lim_{x\to 0^+} f(x) =$ h. $f(1) =$ i. $f(-1.6) =$

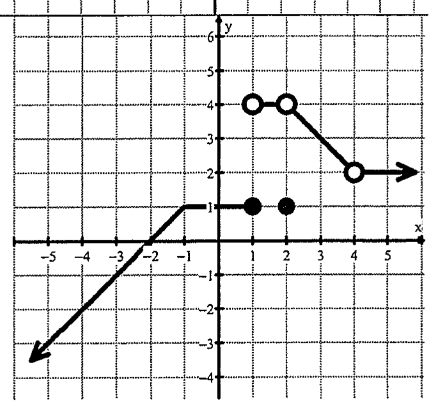


4.

a. $\lim_{x\to -1^-} f(x) =$ b. $f(2) =$ c. $\lim_{x\to 2} f(x) =$

d. $\lim_{x\to -1} f(x) =$ e. $f(4) =$ f. $\lim_{x\to 1^-} f(x) =$

g. $\lim_{x\to -1^+} f(x) =$ h. $f(1) =$ i. $\lim_{x\to 4} f(x) =$

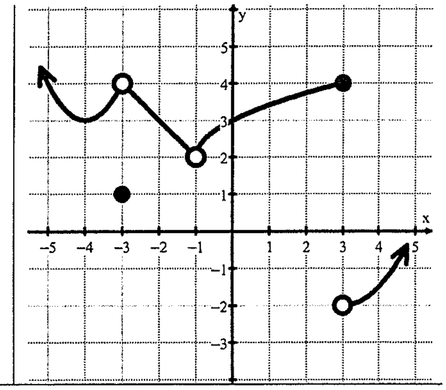


5.

a. $\lim_{x\to 3^-} f(x) =$ b. $f(-1) =$ c. $\lim_{x\to -3} f(x) =$

d. $\lim_{x\to -1} f(x) =$ e. $f(-3) =$ f. $\lim_{x\to 3^+} f(x) =$

g. $f(3) =$ h. $\lim_{x\to 0} f(x) =$ i. $f(-4) =$



6. Sketch a graph of a function f that satisfies all of the following conditions.

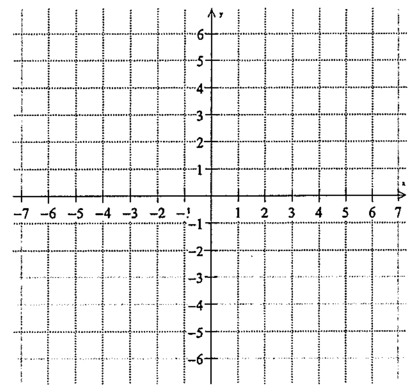
a. $f(-2) = 5$

b. $\lim_{x\to -2} f(x) = 1$

c. $\lim_{x\to 4^+} f(x) = 3$

d. f is increasing on $x < -2$

e. $\lim_{x\to 4^-} f(x) < \lim_{x\to 4^+} f(x)$



7. Sketch a graph of a function g that satisfies all of the following conditions.

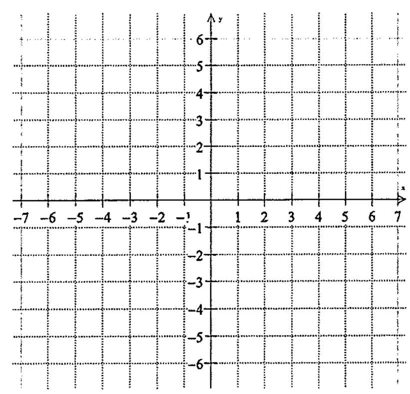
a. $g(1) = 3$

b. $\lim_{x\to 1} g(x) = -2$

c. $\lim_{x\to -3^+} g(x) = 5$

d. g is increasing only on $-5 < x < -3$ and $x > 1$

e. $\lim_{x\to -3^-} g(x) > \lim_{x\to -3^+} g(x)$



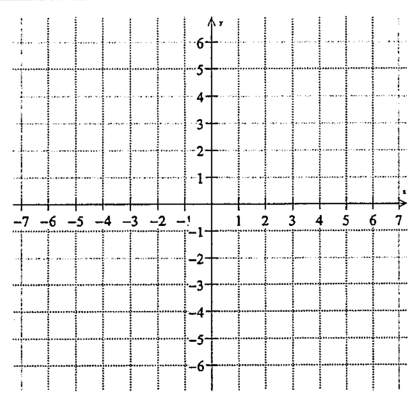
8. Sketch a graph of a function h that satisfies all of the following conditions.

a. $\lim_{x\to 3} h(x) = h(-2) = 1$

b. $h(3)$ is undefined.

c. $\lim_{x\to -2^-} h(x) < \lim_{x\to -2^+} h(x)$

d. h is constant on $-2 < x < 3$ and decreasing everywhere else.



Calculus

Write your questions and thoughts here!

1.2 Limits Analytically

Name:____________________

Notes

Recall: What is a limit?

Finding a limit:

1.

2.

   a.

   b.

3.

Direct Substitution		Factor and Cancel	
1. $\lim_{x \to -1} (x^2 + 2x - 4)$	2. $\lim_{x \to 2} \sqrt{3x - 2}$	3. $\lim_{x \to 0} \frac{4x^2 - 5x}{x}$	4. $\lim_{x \to -7} \frac{2x^2 + 13x - 7}{x + 7}$

Rationalize	Two variables
5. $\lim_{x \to 5} \frac{\sqrt{x + 4} - 3}{x - 5}$	6. $\lim_{h \to 0} \frac{(x + h)^2 - 3(x + h) - (x^2 - 3x)}{h}$

Write your questions and thoughts here!

1.2 Limits Analytically

Notes

Piecewise defined functions and limits

$$f(x) = \begin{cases} \sqrt{11-x}, & x < -5 \\ \dfrac{x+3}{5-x^2}, & x \geq -5 \end{cases}$$

7. $\lim_{x \to -5^-} f(x) =$

8. $\lim_{x \to -5^+} f(x) =$

9. $\lim_{x \to -5} f(x) =$

$$g(x) = \begin{cases} \sqrt{10-x^2}, & x < -1 \\ \dfrac{26-5x^2}{7} & -1 < x \leq e \\ \ln x^3, & x > e \end{cases}$$

10. $\lim_{x \to -1} g(x) =$

11. $\lim_{x \to e^+} g(x) =$

12. $\lim_{x \to e} g(x) =$

Special Trig Limits:

$$\lim_{x \to 0} \frac{\sin x}{x} = \quad \text{or} \quad \lim_{x \to 0} \frac{x}{\sin x} =$$

$$\lim_{x \to 0} \frac{1-\cos x}{x} = \quad \text{or} \quad \lim_{x \to 0} \frac{\cos x - 1}{x} =$$

13. $\lim_{x \to 0} \frac{\sin 3x}{x}$

14. $\lim_{x \to 0} \frac{\tan 4x}{8x}$

15. $\lim_{x \to 0} \frac{\cos^2 x - 1}{x(\cos x + 1)}$

Now summarize what you learned!

1.2 Limits Analytically

Calculus

Name: ____________________

Practice

Evaluate each limit.

1. $\lim_{x\to 2}(x - x^2)$

2. $\lim_{x\to 5}(x + 1)^2$

3. $\lim_{x\to 1}\frac{x^2-5x}{x-1}$

4. $\lim_{x\to 1}\frac{x^2+x-30}{x-1}$

5. $\lim_{x\to 0}\frac{3x}{\sin x}$

6. $\lim_{x\to 0}\frac{\sin(2x)}{3x}$

7. $\lim_{x\to 0}\frac{\sqrt{x+7}-\sqrt{7}}{x}$

8. $\lim_{x\to 7}\frac{\sqrt{x+9}-4}{x-7}$

9. $\lim_{x\to -2}(3x^2 - x + 1)$

10. $\lim_{x\to 3}(2x^2 + 5x - 6)$

11. $\lim_{x\to -7}\frac{2x^3+11x^2-21x}{x^2+7x}$

12. $\lim_{x\to 8}\frac{x^2+2x-80}{x-8}$

13. $\lim_{x\to 0}\frac{\sqrt{x+9}-3}{x}$

14. $\lim_{x\to 0}\frac{\sqrt{x+11}-\sqrt{11}}{x}$

15. $\lim_{x\to 5}\sqrt{4x - 9}$

16. $\lim_{x \to -1} \sqrt{3 - x}$

17. $\lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

18. $\lim_{h \to 0} \frac{5\sqrt{x+h} - 5\sqrt{x}}{h}$

19. $\lim_{x \to \frac{1}{3}} \frac{6x^2+13x-5}{3x-1}$

20. $\lim_{x \to 0} \frac{7x^2+x}{x}$

21. $\lim_{x \to 2} \frac{\sqrt{5x-6}}{x}$

22. $\lim_{x \to \frac{\pi}{2}} \tan\left(\frac{x}{2}\right)$

23. $\lim_{x \to 1} 3$

24. $\lim_{x \to -3} 14$

25. $\lim_{x \to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x}$

26. $\lim_{x \to 0} \frac{\frac{1}{(x+2)^2} - \frac{1}{4}}{x}$

27. $\lim_{x \to 0} (-2)$

28. $\lim_{x \to 1} \frac{\sqrt{x+5} + \sqrt{6}}{x}$

29. $\lim_{x\to 0} \frac{x^2+2x-8}{x-4}$

30. $\lim_{x\to -2} \frac{x^2-4x-10}{x}$

31. $\lim_{x\to 0} \frac{3x^2+5x}{x}$

32. $\lim_{x\to 4} \frac{5x^2-21x+4}{x-4}$

33. $\lim_{x\to \frac{1}{2}} \frac{1-x-2x^2}{2x-1}$

34. $\lim_{x\to \pi} \cos x$

35. $\lim_{x\to \frac{\pi}{8}} \sin(4x)$

36. $\lim_{x\to 2} \frac{x^2+6x-16}{2-x}$

37. $\lim_{x\to 5} \frac{2x^2-17x+35}{5-x}$

38. $\lim_{x\to 0} \frac{(1-\cos^2 x)\sin x}{x^2}$

39. $\lim_{h\to 0} \frac{(x+h)^2+6(x+h)-(x^2+6x)}{h}$

40. $\lim_{h\to 0} \frac{4(x+h)^2-5(x+h)-2-(4x^2-5x-2)}{h}$

3.1 Power Rule

NOTES

CALCULUS

Write your questions here!

Notation

f' $f'(x)$ y' $\frac{dy}{dx}$

POWER RULE

$$x^n =$$

Find the derivative of the following.

$f(x) = 3x^7 - 4x^5 - \frac{1}{3}x^3 + x^2 - 3x + 7$

$y = 2x^{-3} + 4x + \pi$

Rewrite and then take the derivative.

$y = \sqrt[3]{x^7} - \sqrt{x} + 2\sqrt[5]{x^2}$

$g(x) = \frac{1}{x} + \frac{4}{x^2} - \frac{1}{(3x)^2}$

$f(x) = \frac{-16x^2+5x-1}{2x}$

Evaluate

$f(x) = \frac{1}{2}x^4 - 4x^{-2} + e$

Find $f'(3)$

$y = \frac{1}{\sqrt{x}} + 4x$

Find $\left.\frac{dy}{dx}\right|_{x=4}$

Higher Order Derivatives

$f(x) = x^7 - 2x^4 + 5x^2 - 3x + 9$

$f'(x) =$

$f''(x) =$

$f'''(x) =$

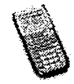
$f^{(4)}(x) =$

$y = \sqrt{x} + x^{-2}$

$\frac{dy}{dx} =$

$\frac{d^2y}{dx^2} =$

Find Derivative on the Calculator



$f(x) = \frac{1}{2}x\sqrt{2x-1}$

$f'(4) =$

$f'(e) =$

$f(\theta) = 1 + \csc\theta$

$f'(\pi) =$

$f'\left(\frac{\pi}{2}\right) =$

Derivative means...

Slope at a point

Given $y = \frac{1}{2}x^4 - x + 2$ find the slope at $x = 2$

Slope of the tangent line

Write the equation of the line tangent to $y = \frac{1}{2}x^4 - x + 2$ at $x = 2$

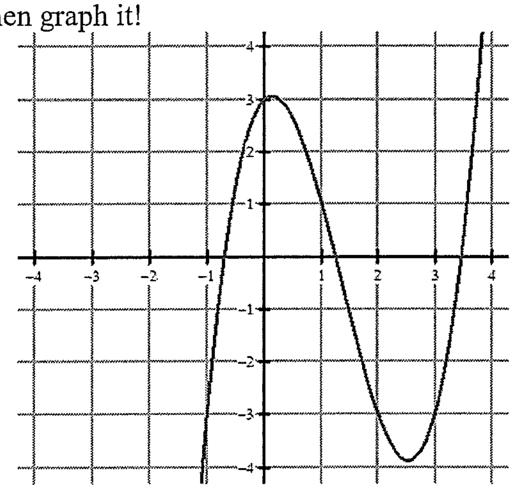
Instantaneous rate of change

What is the instantaneous rate of change at 3 seconds?
$s(t) = -4.9t^2 + 40t + 6$

Normal Line

Write the equation of the normal line at $x = 3$ and then graph it!

$f(x) = x^3 - 4x^2 + x + 3$



Derivative Rules

Constant Rule $\frac{d}{dx}c = 0$

Constant Multiple Rule $\frac{d}{dx}(cu) = c\frac{du}{dx}$

Power Rule $\frac{d}{dx}x^n = nx^{n-1}$

Sum/Difference Rule $\frac{d}{dx}(u \pm v) = \left(\frac{du}{dx} \pm \frac{dv}{dx}\right)$

SUMMARY:

Now, summarize your notes here!

3.1 Power Rule

PRACTICE

Find the derivative of the following.

1. $f(x) = 2x^3 - 4x + 5$	2. $y = 3x^{100} - 2x^8 - 7x$	3. $g(x) = 5x^{-2} - \frac{1}{2}x^4$
4. $h(x) = \frac{x^6}{3} + 6x^{2/3} - 4x^{1/2} + 2$	5. $f(x) = \frac{1}{x^3} + \frac{12}{x}$	6. $y = \frac{3}{x^{-2}} - \frac{1}{(6x)^2}$
7. $f(x) = \sqrt{x} + 3\sqrt[3]{x} + 2$	8. $y = \sqrt[3]{x^2} + 8\sqrt[4]{x^7}$	9. $f(x) = \frac{1}{\sqrt{x}} + \frac{3}{6x}$
10. $f(x) = \frac{1}{\sqrt{x}} + \frac{3}{\sqrt[5]{x^2}}$	11. $s(t) = -16t^2 + 40t + 5$	12. $y = \pi x^2 - \pi$
13. $V(r) = \frac{4}{3}\pi r^3$	14. $f(x) = \frac{2x^3+4x-5}{x}$	15. $g(x) = \frac{6x^3+4x^2-9x}{3}$

Find the derivatives of the following.

16. $f(x) = 3x^7 - 4x^3 + 5x + 7$ $f'(x) =$ $f''(x) =$ $f'''(x) =$ $f^{(4)}(x) =$	17. $y = 4\sqrt{x} + e$ $\frac{dy}{dx} =$ $\frac{d^2y}{dx^2} =$	18. $y = \frac{1}{x^3} - \frac{1}{2}x^4 + ex^2$ $y' =$ $y'' =$ $y''' =$

Given $f(x) = 3x^2 - x + 2$, $g(x) = \frac{1}{x^3} + e^2$, and $h(x) = \sqrt{x}$, find the following.

19. $f'(2) =$	20. $g'''(-3) =$	21. $2h''(4) =$
22. Find the slope of $f(x)$ at $x = 3$.	23. At what value of x is $f'(x) = 0$?	24. What is the slope of the tangent line of $h(x)$ at the point (16, 4) ?

Find the equation for the slope of the line tangent to the given function.

25. $f(x) = 2\sqrt{x} - \pi^2$	26. $y = -2x^3 + \frac{1}{2}x^2 - 7x + 5$	27. $g(x) = \frac{1}{x^2} - \frac{1}{2x}$

Is the slope of the tangent line positive, negative, or zero at the given point?

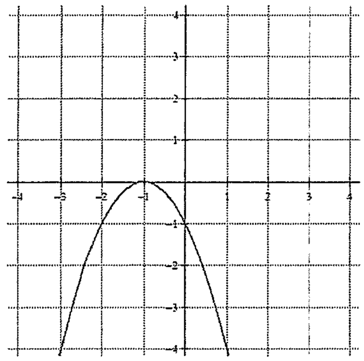
28. $f(x) = \frac{4x^3 - 16x^2}{2x}$ at $x = 2$	29. $y = 2x^4 + 5x^3$ at $x = -2$	30. $g(x) = 3\sqrt[3]{x^5} - 4x^{-1}$ at $x = 8$

Write the equation of the tangent line and the normal line at the point given.

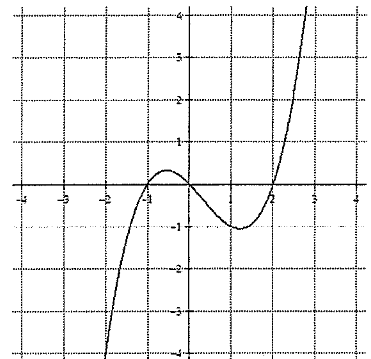
31. $f(x) = 3\sqrt{x} + 4$ at $x = 4$	32. $y = \frac{x^2 + 3x - 4}{2}$ at $x = 8$

The function is graphed below. Write the equation of the tangent line at the given point and graph it.

33. $f(x) = -x^2 - 2x - 1$ at $x = -2$

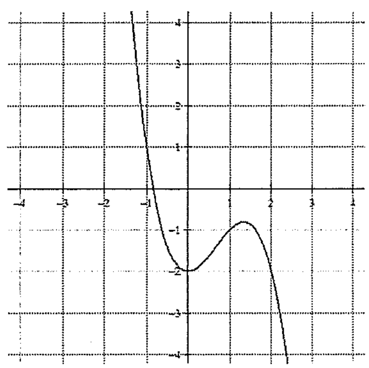


34. $y = \frac{x^3}{2} - \frac{x^2}{2} - x$ at $x = 1$

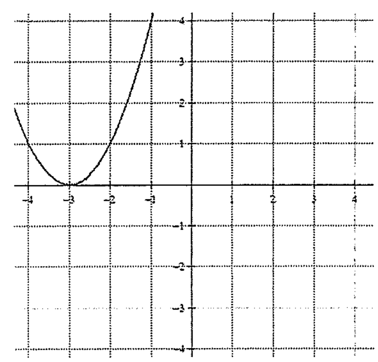


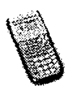
The function is graphed below. Write the equation of the normal line at the given point and graph it.

35. $f(x) = -x^3 + 2x^2 - 2$ at $x = 2$



36. $y = x^2 + 6x + 9$ at $x = -2$





You will need to use a graphing calculator for 37-42

Use the graph to find the derivative of the function at the given value. Round to nearest thousandth.

37. $f(x) = \frac{x^2+1}{x-2}$ at $x = 6$

38. $y = e^x$ at $x = -1$

39. $f(\theta) = 2\sin\theta$ at $\theta = \frac{\pi}{2}$

Write the equation of the tangent line at the point given and sketch the graph. Round to nearest thousandth.

40. $f(x) = -\sqrt{3x+4}$ at $x = 5$

41. $y = \ln(x) + 4$ at $x = e$

42. $f(\theta) = \csc\theta + 1$ at $\theta = \frac{\pi}{4}$

3.2 Product and Quotient Rule

CALCULUS

Write your questions here!

Find the derivative.

$f(x) = (x + 4)(2x - 5)$

PRODUCT RULE

$\frac{d}{dx}(uv) =$

Find the derivative of the following.

$f(x) = (3x^2 + 2x - 3)(x - 1)$

$y = (2x^{-3} + 4x + \pi)(4x + 1)$

Evaluate

$f(x) = \sqrt{x}(3x^2 - 3)$

Find $f'(4)$

Find the derivative.

$f(x) = \frac{x - 5}{2x + 1}$

QUOTIENT RULE

$\frac{d}{dx}(uv) =$

Find the derivative of the following.

$$f(x) = \frac{3x + 1}{2x^2}$$

$$y = \frac{2x^2}{3x + 1}$$

Horizontal Tangents

Find all horizontal tangents for $y = \frac{2x^2}{3x+1}$

Find $f'(4)$ given the following:

$$g(4) = 3 \text{ and } g'(4) = -2$$
$$h(4) = -1 \text{ and } h'(4) = 5$$

$f(x) = g(x) - h(x)$ $\quad$ $f(x) = h(x) + 2$ $\quad$ $f(x) = g(x) + 2h(x)$

$f(x) = \frac{h(x)}{g(x)}$ $\quad$ $f(x) = g(x)h(x)$

SUMMARY:

Now, summarize your notes here!

3.2 Product and Quotient Rule

PRACTICE

Find the derivative of the following.

1. $f(x) = \frac{5x-2}{x^2+1}$

2. $g(x) = (2x+1)(x^3-1)$

3. $y = (3x^2 - 2x)(x^2 + 3x - 4)$

4. $h(x) = \frac{6x^2+3x-5}{3x}$

5. $f(t) = \frac{t+1}{\sqrt{t}}$

6. $f(r) = r^2(5r^3 + 3)$

Find the derivatives of the following.

7. $y = \frac{x}{x-1}$

$\frac{dy}{dx} =$

$\frac{d^2y}{dx^2} =$

8. $y = x^{-2}(ex^3 + 3)$

$y' =$

$y'' =$

Given $f(x) = (x^2 - 5)(3x + 2)$, find the following.

9. $f'(2) =$

10. Find the slope of $f(x)$ at $x = -3$.

11. What is the slope of the tangent line of $f(x)$ at the point (4, 48) ?

Is the slope of the tangent line positive, negative, or zero at the given point?

12. $f(x) = \frac{2-\frac{1}{x}}{x-3}$ at $x = 4$

13. $g(x) = (x + 1)^2$ at $x = -4$

Determine the x-values (if any) at which the function has a horizontal tangent line.

14. $f(x) = \frac{4x^3 - 10x^2}{2x}$

15. $g(x) = \frac{x^2}{x+1}$

Write the equation of the tangent line and the normal line at the point given.

16. $f(x) = \frac{x-1}{x+1}$ at $x = 2$

Find $f'(2)$ given the following.

$g(2) = 3$ and $g'(2) = -2$

$h(2) = -1$ and $h'(2) = 4$

17. $f(x) = 2g(x) + h(x)$

18. $f(x) = 4 - h(x)$

19. $f(x) = \frac{g(x)}{h(x)}$

20. $f(x) = g(x)h(x)$

Calculus

Write your questions and thoughts here!

6.1 Implicit Differentiation

Name:____________________

Notes

Recall:

Explicit equation | Implicit equation

Finding the derivative **explicitly**: $y^2 + 3x = 5x^3$

When you can't isolate y in terms of x (or if solving for y makes taking the derivative CRAZY), then you want to take the derivative implicitly.

Implicit Differentiation Example: Find $\frac{dy}{dx}$ for $y^2 + 3x = 5x^3$

Step 1: Take the derivative normally. Each time a "y" is involved, include a $\frac{dy}{dx}$.

Step 2: Gather all terms with $\frac{dy}{dx}$ on the left side, everything else on the right.

Step 3: Factor out the $\frac{dy}{dx}$ if necessary to create only one $\frac{dy}{dx}$ term.

Step 4. Solve for $\frac{dy}{dx}$.

2. $y^3 - 2x = x^4 + 2y$

3. $3x^2 + 4xy^2 - 5y^3 = 10$

6.1 Implicit Differentiation

Notes

Write your questions and thoughts here!

Derivative at a point – implicit differentiation.

4. Find the equation of any tangent line for $x^2 + y^2 = 4$ at $x = 1$.

2nd Derivative – Implicit Differentiation:

Finding the 2nd derivative implicitly is a little trickier than finding it explicitly. Once you have done a few, you'll see it's just a matter of algebraic substitution.

5. Find $\frac{d^2y}{dx^2}$ for $\cos y = 2x^2$

Now summarize what you learned!

6.1 Implicit Differentiation

Calculus Name: ____________________ **Practice**

Find $\frac{dy}{dx}$.

1. $4 = 5x^2 + 2y^3$	2. $5y^2 + 3 = x^2$	3. $3x = y^3 + 4$
4. $x^2 = 4y^3 + 5y^2$	5. $(4y^3 + 4)^2 = 3x^2$	6. $2x^3 = (3y^3 + 4)^2$
7. $-3y + y^3 = 5x$	8. $5x^3 - 2y = 5y^3$	9. $\sin(x + y) = 2x$
10. $4x + 1 = \cos y^2$	11. $3x^2 - 6y^2 + 5 = 9y - 3x$	12. $y^2 - 7y + x^2 - 4x = 10$

13. $e^{y^3} = x^3 + 1$

14. $5x^2 - e^{4y^2} = -6$

15. $\ln(4y^3) = 5x + 3$

16. $x^3 + 1 = \ln(3y^7)$

17. $x^3 + y^3 = 6xy$

18. $x^3 - 3x^2y^2 = 3y^3$

For 19-23, use implicit differentiation to find $\frac{d^2y}{dx^2}$.

19. $xy = -3$

20. $x^2 + y^2 = 8$

21. $y^2 = 5x^2 - 3x$

22. $y^3 = x^2 - 4$

23. $y^2 + 3y = 4x - 5$

Find the slope of the tangent line at the given point.

24. $2 = 3x^4 + xy^4$ at $(-1, 1)$

25. $x^2 - y^2 = 27$ at $(6, -3)$

26. $x \ln y = 4 - 2x$ at $(2, 1)$

27. $(x - y)^2 - 4x = 20y$ at $(4, 2)$

Write an equation of the line tangent to the curve at the given point.

28. $x^2 + y^2 + 19 = 2x + 12y$ at $(4, 3)$

29. $x \sin 2y = y \cos 2x$ at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

30. Find the points on the curve $x^2 + 2y^2 = 8$ where the tangent line is parallel to the x-axis.

31. Find the point(s) where the following graph has a **vertical** tangent line. $x + y = y^2$

Test Prep

6.1 Implicit Differentiation

1. If $x + \sin y = \ln y$, then $\frac{dy}{dx} =$

(A) $y + y\cos y$ (B) $\frac{y+\cos y-1}{y}$ (C) $\frac{1-y}{y\cos y}$

(D) $\frac{y}{y\cos y+1}$ (E) $\frac{y}{1-y\cos y}$

2. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0, 10)$?

(A) One (B) Three (C) Four (D) Five (E) Seven

3. A curve is generated by the equation $x^2 + 4y^2 = 16$. Determine the number of points on this curve whose corresponding tangent lines are horizontal.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4